Low-order aberration coefficients of systems with freeform surfaces

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Motivation

• Interest in miniaturizing reflective optical systems for smaller platforms

• Additional degrees of freedom are needed in the lens design to maintain (or increase)
  – Field-of-View
  – Aperture Size (or Numerical Aperture)
  for the smaller packages

• Use freeform mirrors

Can we get design insights from aberration theory?
Wave Aberration Function

• For rotationally symmetric systems, Taylor expand imaging properties about axial ray:

\[ W(h, \rho) = w_d \rho^2 + w_1 \rho^4 + w_2 h\rho_y\rho^2 + w_3 (h\rho_y)^2 + w_4 h^2\rho^2 + w_5 h^3\rho_y + O(5) \]

• Transverse Ray Errors

\[ \varepsilon_x \propto \frac{\partial W}{\partial \rho_x} \quad \varepsilon_y \propto \frac{\partial W}{\partial \rho_y} \]
Wave Aberration Function

• For Systems with a Plane of Symmetry, Taylor expand imaging properties about ray from center of object that passes through center of stop (Base Ray):

\[ W(h, \rho) = \frac{1}{2}(w_{dx} \rho_x^2 + w_{dy} \rho_y^2) \]
\[ + \frac{1}{2}B_{112} \rho_x^2 \rho_y + \frac{1}{6}B_{222} \rho_y^3 \]
\[ + N_{112} h_x \rho_x \rho_y + \frac{1}{2}N_{211} h_y \rho_x^2 + \frac{1}{2}N_{222} h_y \rho_y^2 \]
\[ + \frac{1}{2}L_{112} h_x^2 \rho_y + L_{211} h_x h_y \rho_x + \frac{1}{2}L_{222} h_y^2 \rho_y \]
\[ + O(3) \]  

← First Order

← Second Order; Constant over Field

← Second Order; Linear in Field

← Second Order Distortion

← Higher Order
Sample Optical System

X-Y Polynomial Mirrors

\[ z = \frac{cr^2}{1+\sqrt{1-c^2(\kappa+1)r^2}} + \sum_{i,j} C_{i,j} x^i y^j \]
First Order
First Order

• Focal Length (Anamorphic Imagery)

\[
w_x = 2 \tan(\theta) f_x, \quad w_y = 2 \tan(\theta) f_y
\]
First Order

• Focus (Basal Astigmatism)

\[ \frac{1}{2}(w_d x \rho_x^2 + w_d y \rho_y^2) \]

Actual spot diagrams for sample system modified to possess basal astigmatism.
Second Order

Constant over Field
Second Order—Constant over Field

\[
\frac{1}{2} B_{112} \rho_x^2 \rho_y + \frac{1}{6} B_{222} \rho_y^3
\]

\[
\varepsilon_x \propto B_{112} \rho_x \rho_y
\]

\[
\varepsilon_y \propto \frac{1}{2} B_{112} \rho_x^2 + \frac{1}{2} B_{222} \rho_y^2
\]
Second Order—Constant over Field

• Typically there is a mixture of the two terms: $\frac{1}{2}B_{112}\rho_x^2\rho_y + \frac{1}{6}B_{222}\rho_y^3$
• For example, when $B_{112} = 3B_{222}$

$B_{112} = -B_{222}$

Actual Point Spread Function for Sample System modified to possess these combinations of aberration coefficients.
Second Order—Constant over Field

- Typically there is a mixture of the two terms: \( \frac{1}{2} B_{112} \rho_x^2 \rho_y + \frac{1}{6} B_{222} \rho_y^3 \)
- For example, when \( B_{112} = 3B_{222} \)

\[ B_{112} = -B_{222} \]
Second Order

Linear in Field
Second Order—Linear in Field

\[ N_{112} h_x \rho_x \rho_y + \frac{1}{2} N_{211} h_y \rho_x^2 + \frac{1}{2} N_{222} h_y \rho_y^2 \]

\[ \varepsilon_x \propto N_{112} h_x \rho_y + N_{211} h_y \rho_x \]
\[ \varepsilon_y \propto N_{112} h_x \rho_x + N_{222} h_y \rho_y \]
Second Order—Linear in Field

\[ N_{112} h_x \rho_x \rho_y + \frac{1}{2} N_{211} h_y \rho_x^2 + \frac{1}{2} N_{222} h_y \rho_y^2 \]

\[ \varepsilon_x \propto N_{112} h_x \rho_y + N_{211} h_y \rho_x \]

\[ \varepsilon_y \propto N_{112} h_x \rho_x + N_{222} h_y \rho_y \]
Second Order—Linear in Field

\[ N_{211} \text{ and } N_{222} \]

\[ N_{222} \neq 0 \]

Base Ray

\[ N_{211} \neq 0 \]

\[ X' \]
Second Order—Linear in Field

\[ N_{211} \text{ and } N_{222} \]

\[ N_{222} \neq 0 \]

Base Ray

\[ N_{211} \neq 0 \]

Image Tilt

8.41°

7.97°

Spot Diagrams (exaggerated)
On Image Plane
Second Order—Linear in Field

\[ N_{112} h_x \rho_x \rho_y + \frac{1}{2} N_{211} h_y \rho_x^2 + \frac{1}{2} N_{222} h_y \rho_y^2 \]
Second Order—Linear in Field

\[ N_{112} h_x \rho_x \rho_y + \frac{1}{2} N_{211} h_y \rho_x^2 + \frac{1}{2} N_{222} h_y \rho_y^2 \]
Second Order—Linear in Field

\[ N_{121} h_x \rho_x \rho_y + \frac{1}{2} N_{211} h_y \rho_x^2 + \frac{1}{2} N_{222} h_y \rho_y^2 \]
Second Order—Distortion

\[ \frac{1}{2} L_{112} h_x^2 \rho_y + L_{211} h_x h_y \rho_x + \frac{1}{2} L_{222} h_y^2 \rho_y \]

\[ \varepsilon_x \propto L_{211} h_x h_y \]
\[ \varepsilon_y \propto \frac{1}{2} L_{112} h_x^2 + \frac{1}{2} L_{222} h_y^2 \]

Term that is non-zero: \( L_{112} \)
Second Order—Distortion

Distortion of Sample System

Y Field Angle in Object Space (deg)

X Field Angle in Object Space (deg)

Term that is non-zero:

- $L_{112}$
- $L_{211}$
- $L_{222}$
Second Order—Distortion

- Optimize Mirror Profiles to Control Distortion
Second Order—Distortion

• Optimize Mirror Profiles and Geometry to Control Distortion

All second order aberrations have been corrected
Concluding Remarks
How to control low-order properties

• 2\textsuperscript{nd} order coefficients are nonlinear functions of principal curvatures and tilts of mirrors

• Terms $B_{222}$, $N_{222}$, $L_{222}$ are linear in $C_{0,3}$ (i.e., $\frac{\partial^3 Z}{\partial y^3}$)

• Terms $B_{112}$, $N_{112}$, $N_{211}$, $L_{112}$, $L_{211}$ are linear in $C_{2,1}$ (i.e., $\frac{\partial^3 Z}{\partial x^2\partial y}$)

• Surfaces at the stop only affect $B_{112}$ and $B_{222}$

• Image plane tilt affects:
  – Anamorphism (1\textsuperscript{st} order)
  – Some of the 2\textsuperscript{nd} order linear in field
  – Some of the 2\textsuperscript{nd} order distortion

X-Y Polynomial Mirrors

$$z = \frac{cr^2}{1+\sqrt{1-c^2(\kappa+1)r^2}} + \sum_{i,j} C_{i,j} x^i y^j$$
Thank You